Research Paper

Mortality Improvement
Research Paper

Committee on Life Insurance
Financial Reporting

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Memorandum

To: All Life Insurance Practitioners
From: Tyrone G. Faulds, Chair
      Practice Council
            Edward Gibson, Chair
            Designated Group
Date: September 23, 2010
Subject: Mortality Improvement Research Paper

The Committee on Life Insurance Financial Reporting has drafted this paper to provide support for an updated promulgation for mortality improvement with respect to the valuation of insurance and annuity business, and for changes in the range of margins in the Standards of Practice.

The proposed revised Standards of Practice include alterations to subsection 2350, concerning insurance mortality and annuity mortality, and the paper explains the reasoning behind these planned changes.

Research indicates that to offset the reduction in future mortality fully with an increase in the margin for adverse deviations produces an excessive provision with respect to the insurance mortality assumption. The paper includes a summary of recent research on mortality improvement—featuring the Hardy Report commissioned by the Canadian Institute of Actuaries and the Society of Actuaries, and the Office of the Superintendent of Financial Institutions’ Mortality Projection Study—and guidance on future expected mortality improvement.

It also outlines revised levels of margins for adverse deviations for insurance and annuity policy liabilities.

In accordance with the Institute’s Policy on Due Process for the Adoption of Guidance Material Other than Standards of Practice, this research paper has been prepared by the Committee on Life Insurance Financial Reporting, and has received approval for distribution from the Practice Council on September 9, 2010.

If you have any questions or comments regarding this research paper, please contact Edward Gibson, Chair, Designated Group, at his CIA Online Directory address, edward.gibson@empire.ca.

TGF, EG
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Section 1: General Overview

The primary purpose of this research paper is to provide support for an updated promulgation for mortality improvement (a secular trend toward lower mortality rates) with respect to the valuation of insurance and annuity business. It also provides support for the changes in the range of margins in the Standards of Practice.

According to subsection 2350 of the current Standards of Practice, for insurance mortality:

.06 If the actuary’s best estimate assumption includes a secular trend toward lower mortality rates whose effect is to reduce the policy liabilities, then the actuary would negate that trend by an offsetting increase or decrease in what the actuary would otherwise select as a margin for adverse deviations.

.07 The low and high margins for adverse deviations for the mortality rate per 1,000 would be respectively an addition of 3.75 and 15, each divided by the best estimate curtate expectation of life at the life insured’s projected attained age.

For annuity mortality

.11 It is prescribed that the actuary’s best estimate includes a secular trend toward lower mortality rates as promulgated from time to time by the Actuarial Standards Board.

.12 The low and high margins for adverse deviations would be respectively a subtraction of 5% and 15% of the best estimate.

The proposed revised Standards of Practice are as follows:

For insurance mortality:

.05.1 The actuary would consider the inclusion of mortality improvement (a secular trend toward lower mortality rates) in the best estimate assumption.

.06 If the inclusion of mortality improvement reduces the insurance contract liabilities, then the resulting reduction would be no greater than that developed using prescribed mortality improvement rates as promulgated from time to time by the Actuarial Standards Board. If, at an appropriate level of aggregation, the inclusion of mortality improvement increases the insurance contract liabilities, then the actuary’s assumption would include such improvement. The resulting increase in insurance contract liabilities would be at least as great as that developed using prescribed mortality improvement rates as promulgated from time to time by the Actuarial Standards Board.
The low and high margins for adverse deviations for the mortality rates per 1,000 would be respectively an addition or subtraction, as appropriate, of 3.75 and 15, each divided by the curtate expectation of life at the life insured’s projected attained age. These margins for adverse deviations are applied after mortality improvement.

For annuity mortality:

The actuary’s assumption would include mortality improvement, the effect of which is to increase the insurance contract liabilities, such that the resulting increase would be at least as great as that developed using prescribed mortality improvement rates as promulgated from time to time by the Actuarial Standards Board.

The low and high margins for adverse deviations for the mortality rates would be respectively a subtraction of 2% and 8% of the best estimate.

Research indicates that to offset the reduction in future mortality fully with an increase in the margin for adverse deviation produces an excessive provision with respect to the insurance mortality assumption. In addition, recent studies demonstrate that historical mortality improvement observed on population data is higher than Scale AA (more background on this in a later section and a graph), as adjusted by the Committee on Life Insurance Financial Reporting (CLIFR) in its current guidance.

This document provides a summary of recent research on mortality improvement and guidance on future expected mortality improvement. It also outlines revised levels of margins for adverse deviations for insurance and annuity policy liabilities.

Section 2: Mortality Improvement: Recent Research

It is important to base mortality improvement rates guidance on Canadian experience. The following two studies helped in that way.

2.1 The Hardy Report

In 2004, the Canadian Institute of Actuaries and the Society of Actuaries commissioned Mary Hardy of the University of Waterloo to prepare a report on Mortality Improvement Scales for Canadian Insured Lives¹, referred to as the Hardy Report. The objective of the report was to develop a mortality improvement scale (or scales) to incorporate mortality improvement in product analysis, pricing and valuation of life insurance products of Canadian life insurance companies.

The available Canadian insured lives experience contained no information on causes of death and risk factors (other than smoker status, and this was available only for policies issued after 1981). Also, because insured life experience was only available for the 1982–1983 to 2001–2002 policy years and no smoker information in the ultimate data existed before policy year 1992–1993, it inevitably precluded the estimation of the mortality improvement scale separately for smokers and non-smokers.

To overcome these problems, Canadian population mortality experience was used, covering a far longer period of 81 calendar years (1921 to 2002), and providing a much greater number of exposed risks.
The central rate of mortality for each age, m_x, was used because it is an estimate of the mid-year force of mortality, u_{x+1/2}. A constant force of mortality was assumed over the year of age to give an estimated survival probability p_x = e^{-m_x}. The m_x values were calculated for all ages and calendar years. At very high ages, obtained m_x values were not reliable, and an assumption that probabilities of death are increasing with age at a linearly decreasing rate was used to calculate them.

In the report, the population mortality survival rate at age x, s years in the future, is defined as:

\[ p_{x,s} = e^{-m_x (m_{x,s} / m_x)} = e^{-m_x IS(x,s)} = p_x^{IS(x,s)}, \]

and the improvement scale factor is calculated using the following formula:

\[ IS_{2001}^{Population}(x,s) = e^{b_x (k_{1,s} - k_x)} = e^{b_x s} = e^{w_x s} \]

where

- m_x is the central rate of mortality at age x,
- IS(x,s) is the improvement scale factor m_{x,s}/m_x,
- b is a variable related to age x,
- k is a variable proportional to the number of years projected, then k_{1+s} - k_x = cs
- and w_x = b_x c

Please refer to appendix A of this Research Paper for further background information on these formulae.

Because mortality improvement scales normally used by actuaries are applied to q_x, we can use the following approximation to calculate projected mortality rates:

\[ f_x = 1 - e^{w_x} \]

Then we can derive improved mortality rates at time s as:

\[ q_{x,s} = q_{x,0} (1 - f_x)^s \]

The approximation is higher than the exact model rate of improvement f_x and would be used only when q_x is less than 0.1. Please refer to appendix A for formula development, and appendix B for values of w_x and f_x.

This report also defines a relationship between insured lives ultimate mortality improvement rates and population mortality improvement rates:

\[ IS_{insured}(x,s) = exp(h_{2,x} w_x s) = exp(z_x s) \]

where z_x varies by age and sex.

This last formula was not retained in the report, because ultimate mortality data was only available for years 1992 to 2001, which is too short a duration to determine a correlation with last century of population mortality improvement.
Features and caveats of the model include the following:

- A stochastic approach was chosen to provide a holistic picture of longevity risk.

- A purely statistical approach was taken. Data over an 80-year period has been used to model the future. Female mortality improvement rates are higher than those for males, even if male mortality improvement rates have been higher than for females over the 1981-2001 period, according to the OSFI Study, as presented in section 2.2 below.

- A sudden structural shift could render the historical experience irrelevant to future experience. The possible effects on mortality of obesity or a new miracle cure are some examples of structural shifts.

- Mortality shocks applying to a limited range of ages, such as the effect of AIDS on younger male mortality in the decade around 1986-1995 are, to some extent, smoothed out of the data.

- A part of mortality improvement comes from a decrease of smoking habits during the last 50 years.

2.2 OSFI Mortality Projection Study

The Office of the Superintendent of Financial Institutions (OSFI) has presented a study on general trends of mortality in Canada referred to as the OSFI Study. Jean-Claude Ménard, chief actuary at OSFI, presented results of this study on mortality projection in relation to the social security program in Canada at the 15th International Conference of Social Security Actuaries and Statisticians in Helsinki, Finland, on May 24, 2007. He also presented an updated study at the Living to 100 International Research Symposium in Orlando on January 8, 2008.

In the initial study, historical average annual rates of mortality improvement by sex and age group were as follows:

<table>
<thead>
<tr>
<th>Table 2.1: Average Annual Rate of Mortality Improvement (Males)</th>
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<tr>
<td>----------</td>
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<tr>
<td>00–14</td>
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<tr>
<td>15–64</td>
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<tr>
<td>65–89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2.2: Average Annual Rate of Mortality Improvement (Females)</th>
</tr>
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<tbody>
<tr>
<td>----------</td>
</tr>
<tr>
<td>00–14</td>
</tr>
<tr>
<td>15–64</td>
</tr>
<tr>
<td>65–89</td>
</tr>
</tbody>
</table>
Some features of this study were:

- Rates of mortality improvement for both males and females have significantly decreased for ages below 15, from a level of about 5% per year in the 1970s to about 3% per year in the 1990s.

- For ages 15 to 64, the rates of improvement for females have been relatively stable at about 2% per year. For males, these rates have been stable at 2.5% level over the last 10 years, after a significant increase between the 1970s and 1980s.

The updated study shows the following mortality improvement rates:

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>15–44</td>
<td>2.8%</td>
<td>2.6%</td>
</tr>
<tr>
<td>45–64</td>
<td>2.6%</td>
<td>2.4%</td>
</tr>
<tr>
<td>65–84</td>
<td>1.2%</td>
<td>2.0%</td>
</tr>
<tr>
<td>85–89</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
<tr>
<td>90+</td>
<td>-0.6%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Some features of this study were:

- It was noted that the difference between female and male life expectancy at age 65 has reduced in the last 15 years.

- According to this study, cancer is the main cause of death for those aged 45 to 64, and improvement in mortality rates at those ages will come primarily from medical advances. For those aged 65 to 84, heart disease is the main cause of death, and improvement in mortality rates will come primarily from medical advances and changes in lifestyle.

2.3 Other Studies

There are other studies which cover aspects of mortality improvement in Canada. For example, Louis Adam presented a Canadian pensioners mortality study, covering the years 1967 to 2003, at a session of the November 10, 2005 CIA meeting. The study showed average mortality improvement rates for males and females by age over various periods ending in 2003, for Canada Pension Plan (CPP) and Québec Pension Plan (QPP) pensioners (over age 60) with different income levels.

Features of this study included:

- Recent experience only;
- Pensioners mortality only;
- Related to ages 60 and over;
- Mortality improvement rates for females lower than for males; and
- Recent rates of mortality improvement are higher than in earlier years of the study.
Another study has been done by the CIA subcommittee on mortality. It covered individual annuitant mortality over the period 1991 to 2001, and was communicated to members in November 2005. An update to this study was published in March 2009 with experience for policy years 2001 to 2004, after the background work for this research paper was completed, but the update has not affected the recommendations.

Features of this study included:

- Covered individual annuity experience, for ages 60 and over;
- Recent experience only;
- Mortality improvement rates were higher for males than for females;
- Relatively low improvement rates for females; and
- Insufficient detail by age is available for setting an improvement scale.

Section 3: Proposed Mortality Improvement Scales

All of the studies mentioned in the previous section were reviewed and considered. The Hardy Report was used to derive future mortality improvement rates/scales, partly because it was the most robust research report among all reviewed. The methodology was well documented and actuarially sound. The approach is unbiased and the results can be reproduced as all formulas and the source of data are documented in the paper. Although the SOA commissioned Louis Adam’s research on CPP/QPP mortality experience, no report was published and therefore the data were not used. Consideration was given to results from the CIA Annuitant Mortality Experience Subcommittee’s study and to Louis Adam’s research (presented at two CIA meetings), but the exposures were less credible than those from the Hardy Report.

Some of the considerations in setting these mortality improvement scales are discussed below.

3.1 Smoking Status

This paper did not recommend different mortality improvement scales based on smoking status, mainly due to lack of data; “the sparse data available which is reliably categorized by smoker status renders inappropriate any attempt to separate the smokers and non-smokers” (page 117 of the Hardy Report).

It is possible that smokers and non-smokers may exhibit different patterns of mortality improvement and that the decreasing proportion of smokers in the last decades contributed to overall population mortality improvement over the same period. There is, however, no empirical basis to derive different scales for smokers and non-smokers. By taking a longer observation period (1921 to 2001), the impact of smoking incidence is diluted.

3.2 Preferred Risk Classes

It is possible that individuals within preferred risk classes may exhibit different patterns of mortality improvement. However, there is no empirical basis to derive different scales for the various risk classes.
3.3 Gender

Results from the Hardy Report, using the 1921 to 2001 period as a basis to project future mortality improvement, show female improvement rates that are higher than male improvement rates. The OSFI Study on mortality projections presented in Helsinki in 2007\(^2\) shows that for longer experience periods (1931 to 2001), female mortality improvement rates are higher than male improvement rates, but more recent experience periods (1991 to 2001) show the opposite (Tables 3 and 4 of the report). The 1966 to 2001 experience period yields male and female improvement rates that are quite close to each other. These results show that future mortality improvement derived from past experience is highly dependent on the experience period used and therefore no clear pattern based on gender emerges. Under this circumstance, the working group decided that the use of unisex rates was appropriate.

3.4 Population, Insurance and Annuity Data

The Hardy Report is based on population and life insurance data but not annuity data.

According to the Hardy Report and as illustrated in Graph 3.1, insurance data produced much greater mortality improvement than the population data but confidence intervals are also much greater.

Graph 3.1: The Hardy Report Mortality Improvement
Population vs. Insured
Some advantages of using population data to project future mortality improvement are:

- Mortality trends are not affected by changes in underwriting;
- Data is available for 80 years, versus only 20 years for life insurance (versus 36 years of annuity experience from the Adam study and 10 years of annuity experience from the Subcommittee on Annuitant Mortality studies); and
- It can be used for annuities and life insurance.

Advantages of using insurance data are:

- Reflects differences in socio-economic classes between insured and general population; and
- Reflects differences in underwriting and self-selection between insured and general population.

Given the available information and data available, a proposed base scale with mortality improvement rates that are approximately equal to the observed population mortality improvement rates (from the Hardy Report) is appropriate for both life insurance and annuity business and is the proposed approach.

3.5 Proposed Base Mortality Improvement Scale

As illustrated in Graph 3.2, the proposed aggregate base improvement rates fall between male and female rates for ages 18 and above, from the Hardy Report. Proposed aggregate rates also fall between male and female improvement over the 1931 to 2001, the 1966 to 2001 and 1991 to 2001 periods from the OSFI Study (with the exception of ages 65 to 89 in the 1966 to 2001 period). Appendix C includes a table with the proposed base improvement rates. The prescribed rates referenced by the Standards of Practice are proposed to be the base rates plus margin.
3.6 Prescribed Mortality Improvement Rates

It is intended that the prescribed mortality improvement rates include a measure of conservatism. To determine prescribed rates from the base improvement rates, adjustments could take different forms and would be in opposite directions for annuities and for life insurance (with the exception of death supported life insurance business for which policy liabilities are higher when mortality improvement is applied in the valuation).

Consideration was given to whether it would be possible to determine an adjustment to the base rates that would generate a reserve in the CTE(60) to CTE(80) range but this was found to be very difficult to do in a comprehensive and consistent fashion.

Limiting or increasing the number of years of improvement to achieve a reasonable level of conservatism was considered. In the Executive Summary of the Hardy Report, it is noted that “the application of mortality improvement factors beyond, say, 25 years should be regarded with very great caution”. Therefore, for life insurance where the reflection of mortality improvement decreases liabilities, it is recommended that the mortality improvement scale would apply for no more than 25 years after the valuation date. After 25 years, no further mortality improvement would be projected.

For annuity business and for death supported life business, it is recommended that some level of mortality improvement would apply for all years beyond the valuation date. This is consistent with both the Hardy Report and the OSFI Study in which mortality improvement has been observed over the last 80 years. However, for many blocks of annuity business extending the period over which mortality improvement takes place
would not generate sufficient conservatism given that many annuitants would not be expected to live beyond 25 years. Therefore an adjustment to mortality improvement rates in the first 25 years is needed.

In determining the final recommended prescribed rates, the uncertainty approach outlined in the Hardy Report was retained. The following two scenarios were developed which the actuary would run to determine the minimum valuation assumption. The first scenario would be expected to apply in situations where the reflection of mortality improvement decreases liabilities and the second scenario where the effect is to increase liabilities.

1. Mortality improvement would be projected for 25 years only from the valuation date using 50% of the base mortality improvement rates as described above. After 25 years no further mortality improvement would be reflected.

2. Mortality improvement would be projected for all future years using 150% of the base mortality improvement rates as described above for 25 years, and 100% of the base mortality improvement rates as described above thereafter.

The prescribed mortality improvement rates would be the rates from the mortality improvement scenario producing the higher liability, determined at an appropriate level of aggregation. It would be inappropriate to aggregate annuities with life insurance business.

The following graph shows the projected base mortality improvement rates and the rates generated from scenarios 1 and 2 for a life aged 35 at the valuation date.

**Graph 3.3: Prescribed Mortality Improvement Rates**

The following graph shows the mortality improvement rates and the rates generated from scenario 2 for an annuitant aged 55 at the valuation date, compared against the modified AA scale.
Section 4: Margin for Adverse Deviations

4.1 Margin on the Base Assumption

The mortality margins for adverse deviations (MfADs) are currently $3.75/e_x$ to $15/e_x$ for life insurance and 5% to 15% for annuities. In a CALM environment, it would be desirable to have a unified approach to mortality margins. However, because of the work involved and the impending change to International Financial Reporting Standards (IFRS), we recommend that the MfAD formats remain unchanged at this point.

The development of the life insurance margin is described in the CIA paper *Provision for Adverse Deviations* effective January 1, 1990, published March 1990. Under reserve methods such as the 78 Canadian or the net level premium, applying a flat margin could actually reduce rather than increase the policy liabilities. The $k/e_x$ margin approach has the desirable characteristic of being inversely related to the expected assumption. Therefore, it has been recommended that the mortality MfADs range remain at $3.75/e_x$ to $15/e_x$ for life insurance, with the sign in the appropriate direction for developing a positive margin.

The $e_x$ component of the MfAD for the valuation of life insurance at attained age $x$, could be calculated as follows:

$$e_x = (1 - q_x) + (1 - q_x) \times (1 - q_{x+1} \times (1 - Mimp_{x+1})) + (1 - q_x) + (1 - q_{x+1} \times (1 - Mimp_{x+1})) \times (1 - q_{x+2} \times (1 - Mimp_{x+2})^2 + \ldots$$
The calculation of the life expectancy \( t \) years later would then be:

\[
e_{x+t} = (1 - q_{x+t} \times (1 - Mimp_{x+t})') + (1 - q_{x+t} \times (1 - Mimp_{x+t})') \times (1 - q_{x+t+1} \times (1 - Mimp_{x+t+1})') + \ldots
\]

where

- \( e_{x+t} \) is the life expectancy at age \( x+t \)
- \( q_{x+t} \) is the best estimate mortality rate, excluding mortality improvement, at age \( x+t \)
- \( Mimp_{x+t} \) is the best estimate mortality improvement rate at age \( x+t \)

Life annuities tend to be single premium products under which any type of margin would lead to increased liabilities. The main appeal of a flat percentage approach is that it is simple to apply but a major drawback is that it produces margins that are too low at younger ages and too high at older ages. This approach does generate conservative margins for most annuity blocks. If the current MfADs for life insurance were applied to annuities, then annuity provisions for adverse deviation (PfADs) would decrease.

For annuities, it is recommended that the margin range for the best estimate be reduced from its current level of 5% to 15%. It is difficult to find a range that is similar to that of life insurance because the MfAD necessary to cover the misestimation of the mean at CTE(60) to CTE(80) can vary widely by age and size of the portfolio. We recommend a range of 2% to 8% as it approximates the resulting PfAD for a single premium immediate annuity for a 60-year-old policyholder (which we view as a reasonable proxy for an annuity portfolio), using the \( k/ex \) formula for life insurance.

### 4.2 Margin on the Improvement Rate Assumption

Section 3.6 outlines the development of the prescribed mortality improvement rates. Given the range of views on mortality improvement, it was thought to be appropriate to prescribe only a minimum valuation assumption and leave it to the actuary to use appropriate judgement in the determination of a best estimate assumption for future mortality improvement, and the associated margin. Given the nature of the underlying assumption, the normal margin range of 5% to 20%, as outlined in paragraph 2350.01 of the Standards of Practice, would not apply with respect to the mortality improvement assumption.

The margin related to mortality improvement would take into consideration uncertainty with respect to the future improvement assumption due to factors such as:

- Dependence of the assumption on the experience period used;
- Historical improvements due to medical advancements and lifestyle improvements that may not affect mortality to the same degree in the future;
- Potential impacts from medical and technological breakthroughs, which could increase the rate of improvement in the future;
- Current lack of smoker/preferred risk splits;
- No clear pattern for gender split; and
• Use of population rather than insurance/annuity data (for example, the AIDS death rate pattern in the insured population has not been the same as in the general population; target markets are different; genetic testing may affect self-selection).

The provision for adverse deviations for mortality improvement risk would then be measured as the excess of the reported policy liability over the policy liability, inclusive of the reflection of the $k/e_x$ (insurance) or percentage of mortality rate (annuities) margin, resulting from the application of the actuary’s best estimate assumption for mortality improvement.

**Section 5: Mortality Improvement Scales for Out-of-Canada Business**

For markets other than Canada, the actuary would select appropriate mortality improvement rates (inclusive of margin) for both life insurance and annuities. These improvement rates would produce a total liability for each of life insurance and annuities, that is at least as large as what would be produced using the prescribed rates used in Canada, unless experience indicates otherwise.

It would not be unusual for the mortality experience to vary significantly by country, even between developed countries. For instance, the UK has experienced significant cohort effects but this phenomenon has not been found in North America. Life expectancies in China and India are much lower than in Canada but would be expected to narrow the gap as the countries become more developed and richer. This would produce higher rates of mortality improvement.
Appendix A: Related formulas from the Hardy Report

The mortality improvement scale at age $x$ in a time period of $s$ years was defined as:

$$ IS_{2001}^{\text{population}}(x, s) = \left( m_{x,2001+s} / m_{x,2001} \right) $$

If $\log_e(m_{x,t+s}) = a_x + b_x k_{t+s}$ and $(k_{t+s} - k_t) = cs$ and $b_x c = w_x$

then

$$ IS_{2001}^{\text{population}}(x, s) = e^{b_x(k_{t+s} - k_t)} = e^{b_x cs} = e^{w_x s} $$

where all $w_x$ values are graduated values of original ones.

Because $p_{x,0} = e^{-m_x s}$,

we have $p_{x,s} = e^{-m_x (m_{x,s} / m_x)} = e^{-m_x IS_{x,s}} = p_{x,0}^{IS_{x,s}}$.

If we want to find $f_x$ for which $q_{x,1} = 1 - p_{x,1} = 1 - (1 - q_{x,0})^{IS_{x,1}} = 1 - (1 - q_{x,0})^{w_x}$ and using the series

$$(1 - x)^n = 1 - nx + (1/2)n(n - 1)x^2 - ...$$, we have the following approximation:

$$ q_{x,1} = 1 - \left( 1 - e^{w_x} q_{x,0} + (1/2) e^{w_x} (e^{w_x} - 1) q_{x,0}^2 - ... \right) \approx e^{w_x} \left( 1 + (1/2) (1 - e^{w_x}) q_{x,0} \right) q_{x,0} = (1 - f_x) q_{x,0} $$

and we have $f_x \approx 1 - e^{w_x} \left( 1 + (1/2) (1 - e^{w_x}) q_{x,0} \right) = (1 - e^{w_x}) (1 - (1/2) e^{w_x} q_{x,0})$

The approximation $f_x = 1 - e^{w_x}$ is higher than the true $f_x$ and could be used only when $q_{x}$ is less than 0.1. In that case, the error on $f_x$ is less than 5%.
Appendix B: \( w_x \) from the Hardy Report, \( f_x \) values, where \( f_x = 1 - e^{w_x} \).
Appendix C: Proposed Base Mortality Improvement Rates (applies to both females and males, and to both smokers and non-smokers)

<table>
<thead>
<tr>
<th>Attained Age</th>
<th>Base Rates</th>
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<th>Base Rates</th>
<th>Attained Age</th>
<th>Base Rates</th>
</tr>
</thead>
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<td>41</td>
<td>1.95%</td>
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Appendix D: References
