ANALYSIS OF THE MEAN REVERSION OF INTEREST RATES AND THE IMPLICATIONS FOR TRANSFER VALUES

TASK FORCE ON TRANSFER VALUES

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Analysis of the Mean Reversion of Interest Rates and the Implications for Transfer Values

As of October 22, 1999, the CIA Task Force on Pension Transfer Values has tentatively proposed economic assumptions using a two tier\(^1\) approximation to the zero coupon yield curve derived from current market real and nominal bond yields. The purposes of this article are:

- to consider the appropriateness of the proposed interest assumptions for periods in excess of that where yield curve information is available, and
- to consider whether the proposed transfer value basis could be improved based on an analysis of reinvestment risk relating to extended periods of time and the implications of mean reversion of interest rates.

The two tier discount rate approach under review as of October 22, 1999 is consistent with the concepts of:

- duration (i.e. interest rate sensitivity)
- the term structure of interest rates
- equilibrium and arbitrage free pricing theory

A general treatment of these topics is covered in “Financial Economics” recently distributed to all North American actuaries by the Actuarial Foundation.

Government of Canada bonds are generally not available beyond terms of 30 years. The above concepts lead to an extension of the zero coupon yield curve beyond 30 years using the yields applying at the longest available term to maturity. This approach is almost universal in modern pricing theory. For example, a recent article in the July, 1999 edition of the North American Actuarial Journal, entitled “Term Structure Models: A Perspective From the Long Rate” by Yong Yao contends that:

> “the yield curve should level out as term to maturity increases and slopes with large absolute values occur only in the early maturities”.

The article also states that:

> “Dybvig, Ingersoll, and Ross (1996) show that, in frictionless markets having no-arbitrage, the asymptotic long forward and zero coupon rates never fall”.

We note that this article was written to address precisely those situations we are faced with, namely, when the time horizon of cashflows extends beyond the limit of the observable term structure.

Nevertheless, some actuaries suggest that the transfer value basis should be based on fixed ultimate interest assumptions (e.g. 6.00% nominal, and 3.00% or 3.25% real) beyond 15 or 20 years. The argument given in support of that approach is based on the tendency of interest rates to “revert to the mean” and the applicable reinvestment implications.

“Financial Economics” cites the work of Richard Deaves in the context of examining the behaviour of Canadian interest rates, including the predictive performance of mean reversion interest rate models. In fact, Deaves authored a research study for the CIA, “Modelling and Predicting Canadian Inflation and Interest Rates”, published in the 1994 proceedings of the CIA. Using data for 10+ year Canada Bond yields spanning the period 1951-91, Deaves used a mean reversion model to forecast rates for the 15 year period 1976-91. Deaves concluded that:

> “based on the data set of this study, there was essentially no value in forecasting long term interest rates using ARIMA\(^2\) methodology…. one would have been better off predicting by simply using the current rates”.

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\(^1\) That is, a select interest rate for an initial 10 years and an ultimate interest rate thereafter.
Deaves does point out elsewhere that mean reversion is a slow process and that:

“the further out one goes, the less a slow mean reverting autoregressive process will resemble a random walk, and the greater the potential for ARIMA to show its superiority”.

To summarize, research commissioned and published by the CIA finds no evidence that mean reversion based interest rate forecasts are superior to a naïve no change assumption for as long as 15 years out, based on data spanning 1951-1991. On the other hand, it is conceivable that mean reversion may be a factor going out past the observable yield curve, that is, beyond 30 years.

To the best of my knowledge, there is no published research on the strength of mean reversion beyond 15 years. To test out further, I fit an AR (1)\(^3\) model to long term Government of Canada bond yields (annual effective rates). The model is similar to the one used by Deaves, with the primary difference being that I used a lognormal form. The lognormal approach better reflects the stylistic properties of interest rates, and ensures that any stochastic simulation based on this model does not produce negative interest rates. The model is described in more detail in Appendix B.

The model and the available data show that mean reverting process is extremely slow – the AR(1) process is about 97% random walk. This means that the estimated historical mean (6.25% for nominal interest rates) has very limited statistical credibility. The actual mean has a 50% probability of being outside the range 5.11% – 7.63% for nominal interest rates. Although we do not have a lot of historical data available to demonstrate a similar range of real interest rates, it is safe to assume that the mean reverting process for real interest rates is also very slow.

This analysis suggests the possibility of imposing a range on long term interest assumptions adopted for calculating pension transfer values, as follows:

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Real</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>8.5%</td>
<td>4.25%</td>
</tr>
<tr>
<td>Minimum</td>
<td>4.5%</td>
<td>2.25%</td>
</tr>
</tbody>
</table>

Note that the centres of the ranges (6.5% nominal, 3.25% real) correspond to the long term average interest rates (6.25% nominal, 3.00% (est.) real) plus the net economic adjustment of 0.25% (0.50% credit risk less 0.25% expense adjustment).

These maxima and minima would allow our basis to “float” within the given ranges. In effect, we would be consistent with the arbitrage free extension of the yield curve within the limits, while attaching some weight to mean reversion and reinvestment issues outside the range. This is also more consistent with economic theory which would almost certainly reject the concept of a single sustainable (i.e. 6.25% nominal, 3.00% real) equilibrium state for interest rates.

We can proceed with some analysis of the reinvestment issue for a member receiving a commuted value in exchange for a long deferred pension. Reinvestment is not a particular concern if the commuted value is determined at a time when interest rates are close to the long term average or within the expected range. However, reinvestment may be a concern if the commuted value is determined at a time when interest rates are well outside the normal ranges.

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2 Autoregressive integrated moving average
3 Autoregressive function of order 1
As a result, I tested invest/reinvestment strategies in government bonds to reproduce a member’s pension. The main strategy considered involved purchasing and reinvesting in 20 year strip bonds about every five years. Typically, for a 35 year deferred annuity, the duration of the pension can be matched between 20 and 25 years out. The issue of reinvestment risk becomes irrelevant once the liability duration can be matched by an asset with the same duration. A secondary investment strategy considered involved purchase of 30 year government bonds and annual rollover to a new 30 year bond.

These investment strategies were tested in the following interest environments:

- initial nominal interest rate of 15%
- initial nominal interest rate of 3%
- initial real interest rate of 6%
- initial real interest rate of 1.5%

Finally, for each of the investment strategies and for each of the interest rate scenarios, I tested the adequacy of transfer values based on 3 sets of assumptions:

- the current proposed basis subject to the imposed range shown in Table 1 (excluding the credit risk/expense adjustments)
- a 30 year select and ultimate basis, where the ultimate rate after 30 years is the long term average rate (6.25% nominal, 3.00% real)
- a 20 year select and ultimate basis, with ultimate rates in effect under the current Recommendations (6% nominal, 3.25% real).^4

The tests were conducted by comparing the future value of invested assets with the market value of the pension payments at the time when the member can first match the duration of his pension payments with the duration of the bonds utilized.

The results of the tests are provided in Appendix A. More details of the various tests and projections are provided in Appendix C. The main observations for the investment strategy involving 20 year strips traded every 5 years are as follows:

- A transfer value based on the Task Force’s draft recommendations subject to the suggested range for interest rates beyond 30 years produces approximately the desired asset value for each of the interest scenarios considered.
- A transfer value based on a fixed ultimate interest rate after 30 years produces asset values which are too high if interest rates start off higher than normal and too low if interest rates start off lower than normal.
- A transfer value based on the 20 year select and ultimate basis produces asset values which are grossly too large if interest rates start off higher than normal and grossly too low if interest rates start off lower than normal.

For the reinvestment based on 30 year bonds rolled over annually, the transfer values based on the Task Force’s draft recommendations subject to the suggested range for interest rates beyond 30 years produce the best results for 3 of the 4 interest rate scenarios tested, while the 30 year select and ultimate approach with a fixed ultimate rate produces the best result in the other scenario. However, I would not propose that the Task Force base its decision on the rolling 30 year bond investment approach, because the member using such an approach has not minimized the reinvestment risk.

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^4 This corresponds to the current Recommendations with the select period increased to 20 years, as suggested by some members.
Conclusion

I am much more comfortable with the range described here than the 15 or 20 year select and ultimate approaches suggested by some practitioners. Nevertheless, I’m still sceptical of the concept that a transfer/MB value based on a prediction of interest rates many years into the future can be more credible than one based on an arbitrage free extension of a current yield curve, especially in light of Deaves’ research. From a theoretical perspective, commuted values based on the proposed two tier approach produce the best estimates of fair market values. However, if the Task Force wishes to accommodate a reasonable view of mean reversion of interest rates combined with reinvestment risk considerations, the range provided in Table 1 with a 30 year select period is superior to the alternatives that various practitioners have suggested.

Prepared by Martin Den Heyer
October 27, 1999

For the CIA Task Force on Transfer Values

   Brian Burnell
   Marvin Ens (chair)
   Martin Den Heyer
   Minaz Lalani
   Josephine Marks
   Salim Shariff
   Douglas Townsend
Appendix A

Summary of Results of Reinvestment Studies
Based on Assumed Mean Reversion of Interest Rates

Investment Strategy of 20 Years Strips,
Traded About Every 5 Years

35 Year Deferred Pension
Nominal Pension of 10,000 Annually

<table>
<thead>
<tr>
<th>Initial Nominal Interest Rate</th>
<th>Period Until Duration Matching Possible</th>
<th>Interest Rate Upon Duration Matching</th>
<th>Value Upon Duration Matching</th>
<th>Future Asset Value Based on Reinvestment of Transfer Value</th>
<th>Transfer Value Assumption*</th>
<th>Appendix</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.00%</td>
<td>22 years</td>
<td>10.07%</td>
<td>$26,805</td>
<td>$28,642</td>
<td>15% for 30 years, 8.25% after</td>
<td>C-1a</td>
</tr>
<tr>
<td>15.00%</td>
<td>22 years</td>
<td>10.07%</td>
<td>$26,805</td>
<td>$35,991</td>
<td>15% for 30 years, 6.25% after</td>
<td>C-1b</td>
</tr>
<tr>
<td>15.00%</td>
<td>22 years</td>
<td>10.07%</td>
<td>$26,805</td>
<td>$83,749</td>
<td>15% for 20 years, 6.00% after</td>
<td>C-1c</td>
</tr>
<tr>
<td>3.0%</td>
<td>23 years</td>
<td>4.47%</td>
<td>$80,637</td>
<td>$84,954</td>
<td>3.0% for 30 years, 4.00% after</td>
<td>C-2a</td>
</tr>
<tr>
<td>3.0%</td>
<td>23 years</td>
<td>4.47%</td>
<td>$80,637</td>
<td>$66,573</td>
<td>3.0% for 30 years, 6.25% after</td>
<td>C-2b</td>
</tr>
<tr>
<td>3.0%</td>
<td>23 years</td>
<td>4.47%</td>
<td>$80,637</td>
<td>$51,460</td>
<td>3.0% for 20 years, 6.00% after</td>
<td>C-2c</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Nominal Interest Rate</th>
<th>Period Until Duration Matching Possible</th>
<th>Interest Rate Upon Duration Matching</th>
<th>Value Upon Duration Matching</th>
<th>Future Asset Value Based on Reinvestment of Transfer Value</th>
<th>Transfer Value Assumption*</th>
<th>Appendix</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.00%</td>
<td>23 years</td>
<td>4.29%</td>
<td>$83,405</td>
<td>$82,979</td>
<td>6% for 30 years, 4.00% after</td>
<td>C-3a</td>
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<tr>
<td>6.00%</td>
<td>23 years</td>
<td>4.29%</td>
<td>$83,405</td>
<td>$94,417</td>
<td>6% for 30 years, 3.00% after</td>
<td>C-3b</td>
</tr>
<tr>
<td>6.00%</td>
<td>23 years</td>
<td>4.29%</td>
<td>$83,405</td>
<td>$118,854</td>
<td>6% for 20 years, 3.25% after</td>
<td>C-3c</td>
</tr>
<tr>
<td>1.5%</td>
<td>22 years</td>
<td>2.13%</td>
<td>$125,266</td>
<td>$129,247</td>
<td>1.5% for 30 years, 2.00% after</td>
<td>C-4a</td>
</tr>
<tr>
<td>1.5%</td>
<td>22 years</td>
<td>2.13%</td>
<td>$125,266</td>
<td>$113,095</td>
<td>1.5% for 30 years, 3.00% after</td>
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<tr>
<td>1.5%</td>
<td>22 years</td>
<td>2.13%</td>
<td>$125,266</td>
<td>$92,259</td>
<td>1.5% for 20 years, 3.25% after</td>
<td>C-4c</td>
</tr>
</tbody>
</table>

*Prior to adding 0.25% for credit risk net of expenses.

Indicates closest assumptions
Appendix A
(continued)

Summary of Results of Reinvestment Studies
Based on Assumed Mean Reversion of Interest Rates

Investment Strategy of 30 Year Bonds, Rolled Annually

35 Year Deferred Pension
Nominal Pension of 10,000 Annually

<table>
<thead>
<tr>
<th>Initial Nominal Interest Rate</th>
<th>Period Until Duration Matching Possible</th>
<th>Interest Rate Upon Duration Matching</th>
<th>Value Upon Duration Matching</th>
<th>Future Asset Value Based on Reinvestment of Transfer Value</th>
<th>Transfer Value Assumption*</th>
<th>Appendix</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.00%</td>
<td>31 years</td>
<td>9.03%</td>
<td>$70,284</td>
<td>$47,429</td>
<td>15% for 30 years, 8.25% after</td>
<td>C-5a</td>
</tr>
<tr>
<td>15.00%</td>
<td>31 years</td>
<td>9.03%</td>
<td>$70,284</td>
<td>$59,599</td>
<td>15% for 30 years, 6.25% after</td>
<td>C-5b</td>
</tr>
<tr>
<td>15.00%</td>
<td>31 years</td>
<td>9.03%</td>
<td>$70,284</td>
<td>$138,680</td>
<td>15% for 20 years, 6.00% after</td>
<td>C-5c</td>
</tr>
<tr>
<td>3.0%</td>
<td>27 years</td>
<td>4.66%</td>
<td>$93,307</td>
<td>$97,949</td>
<td>3% for 30 years, 4.00% after</td>
<td>C-6a</td>
</tr>
<tr>
<td>3.0%</td>
<td>27 years</td>
<td>4.66%</td>
<td>$93,307</td>
<td>$76,757</td>
<td>3% for 30 years, 6.25% after</td>
<td>C-6b</td>
</tr>
<tr>
<td>3.0%</td>
<td>27 years</td>
<td>4.66%</td>
<td>$93,307</td>
<td>$59,332</td>
<td>3% for 20 years, 6.00% after</td>
<td>C-6c</td>
</tr>
</tbody>
</table>

35 Year Deferred Pension
Inflation Indexed Pension of 10,000 Annually

<table>
<thead>
<tr>
<th>Initial Nominal Interest Rate</th>
<th>Period Until Duration Matching Possible</th>
<th>Interest Rate Upon Duration Matching</th>
<th>Value Upon Duration Matching</th>
<th>Future Asset Value Based on Reinvestment of Transfer Value</th>
<th>Transfer Value Assumption*</th>
<th>Appendix</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.00%</td>
<td>26 years</td>
<td>4.17%</td>
<td>$96,622</td>
<td>$92,493</td>
<td>6% for 30 years, 4.00% after</td>
<td>C-7a</td>
</tr>
<tr>
<td>6.00%</td>
<td>26 years</td>
<td>4.17%</td>
<td>$96,622</td>
<td>$105,243</td>
<td>6% for 30 years, 3.00% after</td>
<td>C-7b</td>
</tr>
<tr>
<td>6.00%</td>
<td>26 years</td>
<td>4.17%</td>
<td>$96,622</td>
<td>$132,481</td>
<td>6% for 20 years, 3.25% after</td>
<td>C-7c</td>
</tr>
<tr>
<td>1.5%</td>
<td>22 years</td>
<td>2.13%</td>
<td>$125,266</td>
<td>$125,058</td>
<td>1.5% for 30 years, 2.00% after</td>
<td>C-8a</td>
</tr>
<tr>
<td>1.5%</td>
<td>22 years</td>
<td>2.13%</td>
<td>$125,266</td>
<td>$109,430</td>
<td>1.5% for 30 years, 3.00% after</td>
<td>C-8b</td>
</tr>
<tr>
<td>1.5%</td>
<td>22 years</td>
<td>2.13%</td>
<td>$125,266</td>
<td>$89,269</td>
<td>1.5% for 20 years, 3.25% after</td>
<td>C-8c</td>
</tr>
</tbody>
</table>

* Prior to adding 0.25% for credit risk net of expenses.

Indicates closest assumptions